

Cube & Hypercube

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A cube is a three-dimensional solid object bounded by six square faces (or facets or sides), with three meeting at each vertex.

The cube can also be called a *regular hexahedron* and is one of the five *Platonic solids*.

The cube is a special kind of *square prism*.

The cube is a special kind of *rectangular parallelepiped*.

The cube is a special kind of *3-sided trapezohedron*.

The cube is dual to the *octahedron*.

The cube has *cubical symmetry* (a.k.a. *octahedral symmetry*).

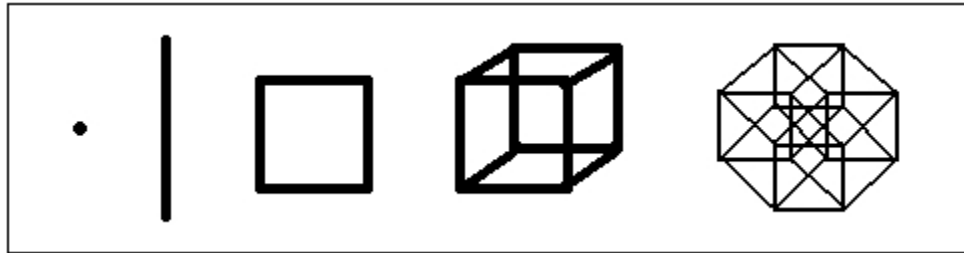
A cube is the three-dimensional case of the more general concept of a *hypercube*, which exists in *any* dimension.

A *hypercube* is an n -dimensional analogue of a square ($n = 2$) and a cube ($n = 3$). It is a closed, compact, convex figure consisting of groups of opposite parallel line segments aligned in each of the space's dimensions, at right angles to each other.

An *n -dimensional hypercube* is also called an *n -cube*. The term "measure polytope" (which is apparently due to Coxeter; see Coxeter, H. S. M., *Regular Polytopes*. 3rd edition, Dover, 1973) is also used but it is rare.

A *unit hypercube* is a hypercube whose side has length one unit. Often, the hypercube whose corners (or vertices) are the 2^n points in \mathbb{R}^n with coordinates equal to 0 or 1 is called "the" unit hypercube.

Translations



- A **point** (see above diagram) is a hypercube of *dimension zero*.
- If one moves this point *one unit length*, it will sweep out a **line segment**, which is a unit hypercube of *dimension one*.
- If one moves this line segment its *length in a perpendicular direction* from itself; it sweeps out a *two-dimensional square*.
- If one moves the square *one unit length in the direction perpendicular to the plane* it lies on, it will generate a *three-dimensional cube*.

This can be generalized to any number of dimensions. For example, if one moves the cube one unit length into the *fourth dimension*, it generates a **4-dimensional unit hypercube** (a **unit tesseract**).

Adapted from:

- <http://en.wikipedia.org/wiki/Cube>
- <http://en.wikipedia.org/wiki/Hypercube>